

BEAM BREAK UP ANALYSIS FOR THE BERKELEY FEMTOSOURCE*

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Abstract

We present a study of the single-bunch beam break up (BBU) instability for a proposed x-ray facility, based on a recirculating linac, to be built in Berkeley. The effects of injection errors, both position and angle, and of misalignments in the linac are investigated. We propose possible methods for limiting the consequent emittance increase.

1 INTRODUCTION

The proposed Berkeley Femtosecond X-Ray Facility [1] (Femtosome) is based on a 600 MeV superconducting recirculated linac. It accelerates up to 2.5 GeV a flat electron beam, which is subsequently used to generate ultra-short X-ray pulses. It is vital to preserve a small vertical emittance throughout the machine since the synchrotron light is produced after the bunch has been rotated by an RF crab cavity.

2 LINAC TRANSVERSE DYNAMICS

The equation describing the transverse displacement $x(s, z)$ of the electrons in an accelerated bunch, as a function of their longitudinal position within the bunch z , can be written in the form [2]:

$$\frac{d}{ds} \left[\gamma(s) \frac{dx}{ds} \right] + k^2(s) \gamma(s) x(s, z) = r_0 \int_z^\infty \rho(z') W_\perp(z' - z) x(s, z') dz' \quad (1)$$

where γ is the relativistic factor, k the focussing strength, r_0 the classical electron radius, ρ the bunch density, W_\perp the transverse wake function per unit length and s indicates the position along the linac. We assume infinitesimally small transverse beam dimensions (a good approximation, when the bunch dimensions are much less than the size of the beam pipe), so that x has to be interpreted as the displacement of the centre of a bunch slice. We also assume the bunch length to be much shorter than the betatron wavelength, therefore the displacement in the RHS of Eq.(1) is not retarded and, finally, we use an average transverse wake function obtained averaging the calculated short range wake of a single cavity [3] over the linac length.

At first we consider the effect associated with a displacement of the electron bunch at the injection into the perfectly aligned linac. Later we extend the analysis to the case of misalignments of the linac RF cavities and cryomodels.

Throughout the paper we compare the results obtained to the output of a simple tracking code, written as a Mathematica notebook.

In order to keep the analytical expressions reasonably simple, we model the linac length as entirely filled with RF cavities (thus neglecting all the drift spaces, accounting for as much as one third of the total length). This causes the wakefield intensity used in the analysis to be larger than the actual value and the average accelerating gradient to be smaller, thus leading to a conservative estimate of the BBU growth.

If there are no focussing elements on the recirculated accelerating section, as is the case in the Berkeley linac, then $k(s) \equiv 0$ and we can rewrite Eq.(1) as:

$$\frac{d^2 x}{du^2} + \frac{1}{u} \frac{dx}{du} = \frac{r_0}{\gamma_i G^2 u} \int_z^\infty \rho(z') W_\perp(z' - z) x(u, z') dz' \quad (2)$$

where $u = 1 + Gs$ is defined with the further assumption of a uniform acceleration over the entire linac length L , $\gamma(s) = \gamma_i(1 + Gs)$ and $G = (\gamma_f / \gamma_i - 1) / L$. γ_i and γ_f are the energy at the beginning and at the end of each pass respectively.

We can solve Eq.(2) using a perturbation technique, beginning with a first order solution $x_{(1)}$:

$$\frac{d^2 x_{(1)}}{du^2} + \frac{1}{u} \frac{dx_{(1)}}{du} = \frac{r_0}{\gamma_i G^2 u} \int_z^\infty \rho(z') W_\perp(z' - z) x_0 dz' \quad (3)$$

with initial conditions: $x(s=0) = x_0$, $x'(s=0) = x'_0$ for all the electrons in the bunch.

This yields:

$$x_{(1)}(z) = x_0 + A_0(z) [u - 1 - \ln u] + x'_0 L \frac{\gamma_i}{\Delta \gamma} \ln u \quad (4)$$

where

$$A_0(z) = x_0 \frac{r_0}{\gamma_i G^2} \int_z^\infty \rho(z') W_\perp(z' - z) dz' \quad (5)$$

Therefore, the relative increment of the transverse displacement at the end of the linac ($s=L$) is:

$$\frac{x_{(1)}}{x_0} = 1 + \frac{A_0}{x_0} [G \cdot L - \ln(1 + G \cdot L)] + \frac{x'_0}{x_0} L \frac{\gamma_i}{\Delta \gamma} \ln(1 + G \cdot L) \quad (6)$$

It is convenient to examine in detail the cases of coordinate and angular displacement at injection separately. For the numerical results the wake for the TESLA RF cavities [3] and a gaussian bunch ($\sigma=1.5$ mm)

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are considered, but the formulas obtained below are valid for an arbitrary bunch distribution.

2.1 Coordinate displacement and no angular displacement: $x(s=0, z)=x_0$, $x'(s=0, z)=0$

If we assume no angle error at injection Eq.(6) becomes:

$$\frac{x_{(1)}}{x_0} = 1 + \frac{A_0}{x_0} [G \cdot L - \ln(1 + G \cdot L)] \quad (7)$$

and $x_{(1)}$ can be easily calculated.

Figure 1 shows the relative displacements, $x_{(1)}/x_0 - 1$, along the bunch at the end of each pass of the linac. The results from our tracking code are also reported.

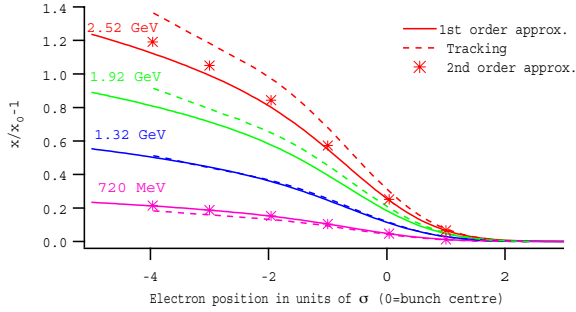


Figure 1. Cumulative transverse displacement (relative to the bunch head), as a function of the electron position within the bunch, after each pass.

We see that after the first pass the bunch tail is displaced by as much as 20% of its initial displacement at injection more than the bunch head.

The second order solution of Eq.(2), at the end of the linac ($u=1+GL$), is

$$\frac{x_{(2)}}{x_0} = \frac{x_{(1)}}{x_0} + \frac{A_1}{x_0} \left[\frac{3}{2} G \cdot L + \frac{1}{4} G^2 L^2 + \left(\frac{3}{2} + G \cdot L \right) \ln(1 + G \cdot L) \right] \quad (8)$$

After numerically evaluating Eq.(8) at several positions along the bunch, shown by stars in Fig. 1, we conclude that we can safely use the first order approximation. Since later passes are equivalent to having a longer linac, we see that the second order approximation at the highest energy begins to be somewhat different from the first order result.

2.2 Angular displacement and no coordinate displacement: $x(s=0, z)=0$, $x'(s=0, z)=x'_0$

In this case Eq.(4) becomes:

$$x_{(1)}(z) = x'_0 L \frac{\gamma_i}{\Delta\gamma} \ln u \quad (9)$$

This, as opposed to Eq.(7), doesn't depend on z and so we need to use the second order solution in this case. We obtain:

$$\begin{aligned} x_{(2)}(z) = & x'_0 L \frac{\gamma_i}{\Delta\gamma} \ln u + \\ & + x'_0 L \frac{\gamma_i}{\Delta\gamma} [(u+1) \ln u - 2(u-1)] \cdot \\ & \cdot \frac{r_0}{\gamma_i G^2} \int_0^\infty \rho(z' + z) W_\perp(z') dz' \end{aligned} \quad (10)$$

At the end of the linac ($u=1+GL$), Eq.(10) is clearer in the following form:

$$\begin{aligned} \frac{x_{(2)}}{x'_0 L} = & \frac{\gamma_i}{\Delta\gamma} \ln(1 + GL) + \\ & + \frac{\gamma_i}{\Delta\gamma} [(2 + GL) \ln(1 + GL) - 2GL] \cdot \\ & \cdot \frac{r_0}{\gamma_i G^2} \int_0^\infty \rho(z' + z) W_\perp(z') dz' \end{aligned} \quad (11)$$

which points out the displacement relative to an injection angle error per unit length, after each linac pass.

Figure 2 shows numerical results for our parameters. The behaviour is qualitatively similar to that of Fig. 1, as it is expected comparing Eqs.(7) and (11).

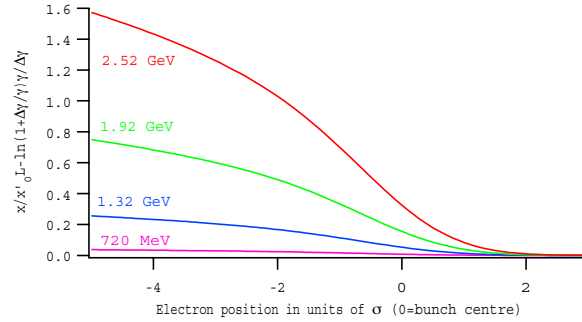


Figure 2. Electron displacement as a function of the position within the bunch (relative to the bunch head).

3 MISALIGNMENTS EFFECTS

In the previous section we have assumed that the accelerator structure is perfectly aligned and the wake field is produced as a consequence of beam injection with a displacement, or angle, error. In this section we study the effect of misalignments of the RF cavities and cryomodules on the transverse dynamics.

To begin with, let's just consider errors in the cavities alignment. Equation (3), which gives the first order approximation for the transverse displacement, has to be only slightly modified to take this into account:

$$\begin{aligned} \frac{d^2 x_{(1)}}{du^2} + \frac{1}{u} \frac{dx_{(1)}}{du} = \\ = \frac{r_0}{\gamma_i G^2 u} \int_z^\infty \rho(z') W_\perp(z' - z) [x_0 + d_c(u)] dz' \end{aligned} \quad (12)$$

Solving Eq.(12) we obtain

$$x_{(1)}(s, z) = x_0 + A_0(z)[Gs - \ln(1 + Gs)] + x'_0 \ln(1 + Gs) + \frac{A_0(z)}{x_0} \sum_{i=0}^{N_{cav}-1} d_{ci} F_i(s) \quad (13)$$

where the expression for the coefficients $F_i(s)$ can be found in [4].

From Eq.(13) we can see that the effect of cavity misalignments is to introduce an additional displacement that, in first approximation, does not depend on the injection coordinates x_0 and x'_0 . It's rms value can be calculated, knowing the misalignment rms value. Choosing $x_0 = x'_0 = 0$, for example, we find:

$$\langle x^2 \rangle^{1/2} = \frac{A_0(z)}{x_0} \sqrt{\sum_{i=0}^{N_{cav}-1} F_i^2(s) \langle d_c^2 \rangle}^{1/2} \quad (14)$$

In calculating Eq.(14), we have assumed no correlation between the alignment errors and this is strictly valid only when analyzing a single pass.

We can also account for the misalignments between the cryomodules in the same way.

With our parameters, 500 and 150 μm [5-6] rms misalignments for the RF cavities and cryomodules respectively, the rms transverse displacement of the electrons at the bunch centre after the first linac pass is about 10 μm .

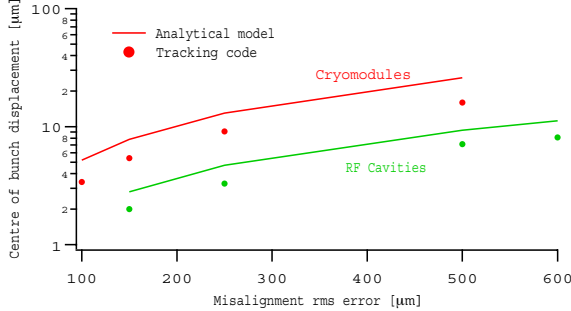


Figure 3. RMS value of the transverse displacement at the bunch centre ($z = 0$) as a function of RF cavities and cryomodules misalignments.

In Fig.3 we compare our analytical results with the tracking code output for several other misalignments values, finding a fairly good agreement.

A direct consequence of the equations found is that it is possible to cancel the effect of misalignments, at the linac exit, by choosing an opportune value for the initial displacement:

$$x_0 = -\sqrt{\frac{\langle d_c^2 \rangle \sum_{i=0}^{N_{cav}-1} F_i^2(L) + \langle d_m^2 \rangle \sum_{i=0}^{N_{mod}-1} G_i^2(L)}{GL - \ln(1 + GL)}} \quad (15)$$

This value can only be measured, after the linac has been assembled and aligned, by direct measurements of the beam profile. For our machine parameters, we expect the required offset to be in the range of 50÷100 μm .

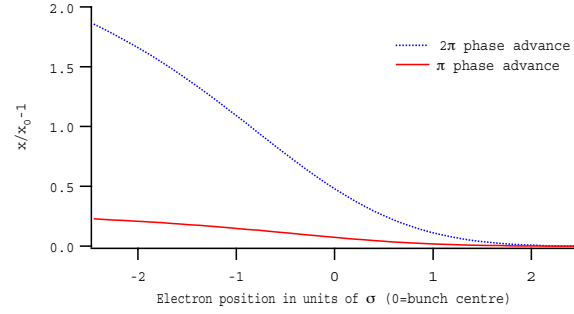


Figure 4. Electron displacement for zero betatron phase advance in all arcs (dotted line) and when the first arc has a π phase advance.

It is also possible to design the the arcs in such a way that the subsequent linac passes will have a canceling effect on the displacement introduced in the first pass. Figure 4 shows the result from the tracking code with a betatron phase advance of π in the first and 2π in the subsequent arcs as opposed to the case of 2π phase advance in all four arcs. The curves reported were obtained running a numerical tracking code with a 500 μm rms displacement error in the cavities and cryomodules and a 50 μrad exit-entrance tilt error in the cryomodules. The improvement can be explained by the fact that changing the sign of the betatron phase along the bunch after the first turn effectively causes the wakefield to reduce the transverse displacement in later passes.

7. CONCLUSIONS

In this paper we describe a study of a single bunch beam break-up instability for recirculating linac based on the TESLA superconducting RF cavities.

We develop the analytical model of the phenomena suitable for the case under study and use it to obtain closed form solutions. Our study suggests that mechanical misalignments of RF cavities and cryomodules are expected to be the strongest source of the instability. We propose to use a particular pattern in the betatron phase advance in successive beam passes through the linac to minimize the beam break up growth.

We also show that it is possible to eliminate the instability in all practical cases of alignment errors by injecting the beam with an orbit offset to be determined by beam-based measurements.

All analytical results were tested with a specially developed tracking code.

6 REFERENCES

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